Homework II - Topic: System Properties

Problem 1

The input-output relationship of a system is given by:

$$y(t) = \int_{-\infty}^{+\infty} (-1)^t e^\tau x(\tau) d\tau$$

a. Is this system stable? Justify your answer.
b. Is this system linear? Justify your answer.
c. Is this system Time invariant? Justify your answer.

Problem 2

The input-output relationship of a system is given by:

$$y(t) = x(t)x(t+1) - 2x(t-2)$$

a. Is this system linear? Justify your answer.
b. Is this system time-invariant? Justify your answer.

Problem 3

Consider a system whose input-output relationship is given by:

$$\frac{dy(t)}{dt} + 4ty(t) = 2x(t)$$

a. Is this system linear? Justify your answer.
b. Is the system causal? Justify your answer.
c. Is the system memory-less? Justify your answer.

Problem 4

Determine whether each of the statements is true or False. You **must** Justify your answer to get a grade

    a. If y(t) is the output of a linear time-invariant system for an input x(t), then y(-t) is the output for the input x(-t).

    b. For an unstable system, every bounded input x(t) yields an output that is not bounded.

c. If x(t) is a periodic signal, then x(t)+x(at) is periodic for any real number a.

Problem 5

The response of an LTI system to a unit step input x(t)= 4u(t) is y(t)= $4(1-e^{-2t})u(t)$. What is the response to an input of $x(t) = 4u(t) - 4u(t-1)$?

Problem 6

    a. Prove whether or not the system defined by: $y(t) = x(t)\cos[x(t)] \cdot \sin[x(t)]$
       Is time invariant or not

    b. Prove if the system defined by $y(t) = [x(t-1)] + 2$ is linear or not

    c. Is the system defined by $y(t) = x\left(\frac{t}{3}\right)$ causal? Why or why not?

Problem 7

A system is defined by the input-output relationship given by:

$$y(t) = x(t) \sin(2t) + 1$$

    a. Is this system linear? Justify your answer.

    b. Is this system time-invariant? Justify your answer?

    c. Is this system stable? Justify your answer.

    d. Is this system causal? Justify your answer.

Problem 8
A system takes an input x(t) and produces the output y(t) given by

$$y(t) = \int_0^t x(\tau)d\tau$$

a. Is the system linear? Justify your answer.
b. Is the system time-invariant? Justify your answer.
c. Is the system stable? Justify your answer.
d. Is the system causal? Justify your work.

Problem 9
Let H denote a continuous system such that the relationship between the input f(t) and the output y(t) is given by the equation

$$y(t) - \frac{1}{2}y(t-1) = tf(t)$$

Is the system linear? Justify your answer

Problem 10
The output y(t) of a continuous-time system is related to its input x(t) by

$$y(t) = \cos[2x(t+1)] + x(t)$$

a. Is the system linear? Justify your answer.
b. Is the system time-invariant? Justify your answer.
c. Is the system causal? Justify your answer.
d. Is the system memoryless? Justify your answer.
e. Is the system stable? Justify your answer.

Problem 11
Consider a continuous-time system which has input of signal x(t) and output of y(t) = x(t)u(t). Is this system time invariant? Justify your answer.